PHYS 110A - HW #5
Solutions by David Pace
Any referenced equations are from Griffiths

[1] Problem 3.12 from Griffiths

Find the potential in the infinite slot of example 3.3 (Griffiths, page 127) if the boundary at x = 0 is composed of two metal strips. One strip extends from $0 \le y \ge a/2$ and is at a constants potential V_0. The other strip extends from $a/2 \le y \ge a$ and is at a constant potential of -V_0.

An illustration of the geometry is given as Figure 3.17 in Griffiths. Set boundary conditions on V(x,y,z). Note that we still have z-symmetry so we will be solving a two dimensional problem, i.e. V = V(x,y).

$(i)\ \ V(0,\ 0 \le y \ge \frac{a}{2}) = V_0 \qquad (ii)\ \ V(0,\ \frac{a}{2} \le y \ge a) = -V_0$

$(iii)\ \ V(\infty, y) = 0 \qquad (iv)\ \ V(x,0) = 0$

$(iv)\ \ V(x,a) = 0$

Solve this two dimensional problem using separation of variables.

The potential is given as $V(x,y) = X(x)Y(y)$ and since there are no charges present the potential at any point is given by the Laplacian:

$$\vec{\nabla}^2 V(x,y) = 0 = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}$$

This simplifies to eq. 3.23: $\quad \frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} = 0$

Each term in eq. 3.23 must be constant, and this leads us to (still following example 3.3):

$$\frac{d^2X}{dx^2} = kX^2 \qquad \frac{d^2Y}{dy^2} = -k^2Y$$

The decision of which term gets the $-k^2$ is made by determining how the field behaves as each variable goes to infinity. The term with the negative sign will be solved in terms of sine and cosine functions and therefore will not go to zero as the variable approaches infinity. The other equation is solved in terms of exponentials and can therefore be made to approach zero as its variable goes to infinity. In this problem the y coordinate is restricted and never goes to infinity. The x coordinate, however, does stretch to infinity in the region we care about and we also know that the potential is zero there from boundary condition (iii). Thus, we choose the equation with x dependence to have the positive k^2.

The X(x) and Y(y) functions are given by:

$$X(x) = Ae^{kx} + Be^{-kx} \qquad\qquad Y(y) = C\sin(ky) + D\cos(ky)$$

The potential is then written as,

$$V(x,y) = \left[Ae^{kx} + Be^{-kx}\right]\Big(C\sin(ky) + D\cos(ky)\Big)$$

Boundary condition (iii) tells us that A = 0. Condition (iv) requires that D = 0. Rewrite the expression for the potential and combine the constants (I also write the new constant as C just so this is similar to the way Griffiths gives examples).

$$V(x,y) = Ce^{-kx}\sin(ky)$$

The constant C is non-zero, boundary condition (v) then requires,

$$V(x,a) = 0 = Ce^{-kx}\sin(ka) \qquad \text{leading to} \quad \sin(ka) = 0$$

$$k = \tfrac{n\pi}{a}\text{where n = positive integer}$$

$$\text{Simplified expression for potential:} \quad V(x,y) = Ce^{\frac{n\pi x}{a}}\sin\left(\frac{n\pi y}{a}\right)$$

The exact solution is given by the sum over all possible solutions (i.e. all possible values of n, which is an integer).

$$V(x,y) = \sum_{n=1}^{\infty} C_n e^{\frac{-n\pi x}{a}}\sin\left(\frac{n\pi y}{a}\right)$$

All that remains is to satisfy boundary conditions (i) and (ii). For example, the plate held at potential V_0 rests between y = 0 and y = a/2 and satisfies the following equation.

$$V(0, 0 < y < \frac{a}{2}) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi y}{a}\right) = V_0$$

Application of Fourier's Trick:

$$\sum_{n=1}^{\infty} C_n \int_0^a \sin\left(\frac{n\pi y}{a}\right)\sin\left(\frac{n'\pi y}{a}\right)dy = \int_0^{\frac{a}{2}} V_0 \sin\left(\frac{n'\pi y}{a}\right)dy - \int_{\frac{a}{2}}^{a} V_0\sin\left(\frac{n'\pi y}{a}\right)dy \tag{1}$$

This is the point where we have deviated from example 3.3. The right hand side of eq. 1 is split into two integrals because the value of the potential is different on each plate. Fortunately, this value is still a constant and may be factored out of both integrals.

The integral on the left side of eq. 1 has already been solved for us.

$$\int_0^a \sin\left(\frac{n\pi y}{a}\right)\sin\left(\frac{n'\pi y}{a}\right)dy = \begin{cases} 0 & \text{for } n' \neq n \\ \frac{a}{2} & \text{for } n' = n \end{cases}$$

$$\text{The left hand side of eq. 1} \quad = \quad \sum_{n=1}^{\infty} \frac{aC_n}{2} \qquad \text{for n'=n}$$

Now solve the right side of eq. 1 where we have just shown n′ = n.

$$\begin{aligned} \text{First integral} \qquad \int_0^{\frac{a}{2}} V_0 \sin\left(\frac{n\pi y}{a}\right)dy &= V_0\left(-\frac{a}{n\pi}\cos\left(\frac{n\pi y}{a}\right)\right)_0^{\frac{a}{2}} \\ &= \frac{-V_0 a}{n\pi}\left[\cos\left(\frac{n\pi}{a}\cdot\frac{a}{2}\right) - \cos(0)\right] \\ &= \frac{-V_0 a}{n\pi}\left[\cos\left(\frac{n\pi}{2}\right) - 1\right] \end{aligned}$$

Note the following:

$$\cos\left(\frac{n\pi}{2}\right) = \begin{cases} 0 & \text{for n = odd} \\ 1 & \text{for n = 0, 4, 8, etc.} \\ -1 & \text{for n = 2, 6, 10, etc.} \end{cases}$$

The other term is:

$$\begin{aligned} \int_{\frac{a}{2}}^a -V_0 \sin\left(\frac{n\pi y}{a}\right)dy &= -V_0\left(-\frac{a}{n\pi}\cos\left(\frac{n\pi y}{a}\right)\right)_{\frac{a}{2}}^{a} \\ &= \frac{V_0 a}{n\pi}\left[\cos\left(\frac{n\pi}{a}\cdot a\right) - \cos\left(\frac{n\pi}{a}\cdot\frac{a}{2}\right)\right] \\ &= \frac{V_0 a}{n\pi}\left[\cos(n\pi) - \cos\left(\frac{n\pi}{2}\right)\right] \end{aligned}$$

$$\cos(n\pi) = \begin{cases} -1 & \text{for n = odd} \\ 1 & \text{for n = even} \end{cases}$$

Take all the possible outcomes for the preceeding two terms and add them up to find,

$$\text{Right side of eq. 1} = \begin{cases} 0 & \text{for n = odd} \\ 0 & \text{for n = 0, 4, 8, etc.} \\ \frac{4aV_0}{n\pi} & \text{for n = 2, 6, 10, etc.} \end{cases}$$

Equate the two sides of eq. 1 and solve for a particular value of C_n

$$C_n = \begin{cases} \frac{8V_0}{n\pi} & \text{for n = 2, 6, 10, etc} \\ 0 & \text{otherwise} \end{cases}$$

Finally, we can insert the value of C_n into the complete solution for the potential. We only take the n = 2, 6, 10, etc. terms because the value of C_n is zero for any other value of n.

$$\boxed{V(x,y) = \sum_{n=2,6,...}^{\infty} \left(\frac{8V_0}{n\pi}\right) e^{\frac{-n\pi x}{a}} \sin\left(\frac{n\pi y}{a}\right)}$$

[2] Problem 3.15 from Griffiths

A cubical box has five of its sides grounded (see Griffiths, Figure 3.23). The sides of the box have length, a, and the top side is held at a potential V_0. Find the potential in the box.

This problem is three dimensional, we will not be able to reduce it. Begin by laying out the boundary conditions for V(x,y,z).

$$(i)\ \ V(a,y,z) = 0 \qquad (ii)\ \ V(0,y,z) = 0$$

$$(iii)\ \ V(x,0,z) = 0 \qquad (iv)\ \ V(x,a,z) = 0$$

$$(v)\ \ V(x,y,0) = 0 \qquad (vi)\ \ V(x,y,a) = V_0$$

Separation of variables is still used to solve Laplace's equation in three dimensions. Let the potential be written as V(x,y,z) = X(x)Y(y)Z(z) and then insert this into Laplace's equation:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Resulting equation: $$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} = 0$$

As with the other problems of this type, each term in the equation above must be constant. The boundaries require that the x and y terms go to zero so we decide to let them be described by the negative constants. Recall that giving them the negative constants will allow us to write them as sinusoidal functions that can be made to be zero on the boundaries.

$$\frac{d^2X}{dx^2} = -k^2 X \qquad \frac{d^2Y}{dy^2} = -l^2 Y \qquad \frac{d^2Z}{dz^2} = (k^2 + l^2) Z$$

This gives the following solutions to each differential equation:

$$X(x) = A\sin(kx) + B\cos(kx) \qquad Y(y) = C\sin(ly) + D\cos(ly)$$

$$Z(z) = Ge^{z\sqrt{k^2+l^2}} + He^{-z\sqrt{k^2+l^2}}$$

Boundary condition (ii) requires that B = 0. Condition (iv) requires that D = 0. Condition (iii) requires that G = -H and allows us to rewrite the z solution:

$$\begin{aligned} Z(z) &= -He^{z\sqrt{k^2+l^2}} + He^{-z\sqrt{k^2+l^2}} \\ &= -H\left[e^{z\sqrt{k^2+l^2}} - e^{-z\sqrt{k^2+l^2}}\right] \end{aligned}$$

Using the identity: $\sinh(\theta) = \frac{1}{2}\left[e^{\theta} - e^{-\theta}\right]$ we can rewrite the Z(z) solution again.

$$\begin{aligned} Z(z) &= -2H\sinh(z\sqrt{k^2+l^2}) \\ &= 2G\sinh(z\sqrt{k^2+l^2}) \end{aligned}$$

Put everything back together (and lump all constants into J):

$$\begin{aligned} V(x,y,z) &= [A\sin(kx)][C\sin(ly)][2G\sinh(z\sqrt{k^2+l^2})] \\ &= J\sin(kx)\sin(ly)\sinh(z\sqrt{k^2+l^2}) \end{aligned}$$

Back to the boundary conditions (letting n, m be positive integers):

$$(i) \qquad V(a,y,z) = 0 = \sin(ka) \quad \rightarrow \quad k = \tfrac{n\pi}{a}$$

$$(v) \qquad V(x,a,z) = 0 = \sin(la) \quad \rightarrow \quad l = \tfrac{m\pi}{a}$$

The solution must then be a sum over all the possible values of n and m.

$$V(x,y,z) = \sum_{n}^{\infty}\sum_{m}^{\infty} J_{n,m}\sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi y}{a}\right)\sinh\left(\frac{\pi z}{a}\sqrt{n^2+m^2}\right) \tag{2}$$

The final boundary to satisfy, (vi), requires that this equation equal the constant V_0 at z = a. The $J_{n,m}$ may be solved for in the same way as in the 2D case, although it requires more algebra. Multiply both sides by the appropriate n′ and m′ factors and then integrate (Fourier's Trick). Start with the right side of eq. 2 evaluated at z = a (just concentrate on the integral first, then add in the $J_{n,m}$ term).

$$\sum_{n,m}^{\infty} J_{n,m} \int_0^a \int_0^a \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{n'\pi x}{a}\right) \sin\left(\frac{m\pi y}{a}\right) \sin\left(\frac{m'\pi y}{a}\right) \sinh(\pi\sqrt{k^2+l^2}) dxdy$$

We have already solved these integrals:

$$\int_0^a \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{n'\pi x}{a}\right) = \begin{cases} 0 & \text{for} \quad n' \neq n \\ \frac{a}{2} & \text{for} \quad n' = n \end{cases}$$

The same holds true for the y integrals and the right side of eq. 2 becomes,

$$\sum_{n,m} J_{n,m} \left(\frac{a}{2}\right) \left(\frac{a}{2}\right) \sinh(\pi\sqrt{k^2+l^2})$$

Return to the left side of eq. 2 , knowing that only n′ = n and m′ = m values are non-zero.

$$\int_0^a \int_0^a V_0 \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi y}{a}\right) dxdxy$$

This has been solved previously as well. Each integral goes as the following.

$$\begin{aligned} V_0 \int_0^a \sin\left(\frac{n\pi x}{a}\right) dx &= -\frac{a}{n\pi} \cos\left(\frac{n\pi x}{a}\right)_0^a \\ &= \frac{-a}{n\pi} [\cos(n\pi) - \cos(0)] \\ &= \begin{cases} 0 & \text{n = even} \\ \frac{2a}{n\pi} & \text{n = odd} \end{cases} \end{aligned}$$

$$\text{Left side of eq. 2} = \begin{cases} \frac{4a^2V_0}{\pi^2 nm} & \text{n and m = odd} \\ 0 & \text{otherwise} \end{cases}$$

Bring it all together for any single case where both n and m are odd (all other cases would result in zeroes anyway).

$$J_{n,m} \frac{a^2}{4} \sinh(\pi\sqrt{n^2+m^2}) = \frac{4a^2V_0}{\pi^2 nm}$$

$$J_{n,m} = \frac{16V_0}{\pi^2 nm}\left[\sinh(\pi\sqrt{n^2+m^2})\right]^{-1}$$

Having solved for $J_{n,m}$ we can write the final solution:

$$\boxed{V(x,y,z) = \sum_{n,m=1,3,5,\ldots}^{\infty} \frac{16V_0}{\pi^2 nm}\left[\sinh(\pi\sqrt{n^2+m^2})\right]^{-1} \sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi y}{a}\right)\sinh\left(\frac{\pi z}{a}\sqrt{n^2+m^2}\right)}$$

[3] Problem 3.17 from Griffiths

(a) Suppose the potential is a constant value V_0 over the surface of a sphere. Use the results of Griffiths' examples 3.6 and 3.7 to find the potential inside and outside the sphere.

(b) Find the potential inside and outside a spherical shell carrying a uniform surface charge density given by σ_0. Use the results of Griffiths' example 3.9.

Part (a)

The solutions to this problem have already been shown to us.

$$V_{inside}(r,\theta) = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta) \qquad \text{Example 3.6, page 139}$$

$$V_{outside}(r,\theta) = \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta) \qquad \text{Example 3.7, page 140}$$

When evaluated at the surface of the sphere each of these equations should return the same value for the potential. In this part of the problem we will determine the constants necessary in order for these values at r = R to match. Begin by looking at the outside solution.

$$V_{outside}(R,\theta) = V_0 = \sum_{l=0}^{\infty} A_l R^l P_l(\cos\theta)$$

Since V_0 is a constant it has no θ dependence and the Legendre polynomial terms must only go up to the l = 0 value. Any other values for l and there would be $\cos\theta$ terms present.

$$V_0 = A_0 R^0 P_0(\cos\theta) = A_0$$

Solving for A_0 was easy, using the integral expression given in ex. 3.6 one could also derive this answer. Now do the same thing for the outside potential expression.

$$V_{outside}(R,\theta) = V_0 = \sum_{l=0}^{\infty} \frac{B_l}{R^{l+1}} P_l(\cos\theta)$$

Only keep l = 0 terms $\qquad V_0 = \dfrac{B_0}{R}$

Now we know the values for all the constants and we can write the final solutions for the potential.

$V_{inside}(r,\theta) = V_0$ constant, as it should be inside a conductor

$V_{outside}(r,\theta) = \dfrac{B_0}{r} = \dfrac{RV_0}{r}$ a 1/r dependence, as we expect away from a source

Part (b)

$\sigma_{r=R} = \sigma_0$

Again, the previous examples have given us the general solutions inside and outside the sphere.

$$V_{insde}(r,\theta) = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta)$$

$$V_{outside}(r,\theta) = \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta)$$

Example 3.9 in Griffiths (page 142) tells us how the potential is discontinuous at the surface.

$$\left(\frac{\partial V_{out}}{\partial r} - \frac{\partial V_{in}}{\partial r}\right)_{r=R} = \frac{-\sigma_0(\theta)}{\epsilon_0} \qquad \text{Eq. 3.82}$$

This simplifies to (explanation in ex. 3.9),

$$\sum_{l=0}^{\infty} (2l+1) A_l R^{l-1} P_l(\cos\theta) = \frac{\sigma_0}{\epsilon_0} \qquad \text{Eq. 3.83}$$

The right side has no θ dependence because σ_0 is constant so the solution must have only the l = 0 term.

$$\frac{A_0}{R} = \frac{\sigma_0}{\epsilon_0} \qquad \rightarrow \qquad A_0 = \frac{R\sigma_0}{\epsilon_0}$$

We have also been given the solution for the B_l constants in terms of the A_l constants.

$$B_l = A_l R^{2l+1} \qquad \text{But only l = 0 terms are non-zero}$$

$$B_0 = A_0 R = \frac{R^2 \sigma_0}{\epsilon_0}$$

Having solved for all of the constants we can write the final solutions.

$$\boxed{V_{inside}(r,\theta) = \frac{R\sigma_0}{\epsilon_0} \qquad V_{outside}(r,\theta) = \frac{B_0}{r} = \frac{R^2\sigma_0}{\epsilon_0}}$$

[4] Problem 3.18 from Griffiths

Given the potential at the surface of a sphere (radius R) as $V_0 = k \ \cos(3\theta)$ find the potential inside and outside of the sphere as well as the charge density on the sphere. Assume there is no charge inside or outside the sphere and that k is a constant.

Notice that V_0 is not given in the usual $\cos(\theta)$ form. Put it in a more recognizable form using the triple angle rule.

$$\cos(3\theta) = 4cos^3(\theta) - 3\cos(\theta)$$

Follow the previous method of evaluating the general solutions for the potential at the surface. Both solutions (inside and outside) should return the given value of the potential at the surface. This time it will not be as simple because we can see that the $cos^3(\theta)$ term requires we go to l = 3 due to the Legendre polynomials. The potential at the surface of the sphere, as given by the solution inside is,

$$V_{inside}(R,\theta) = A_0 + A_1 R\cos(\theta) + A_2 R^2 \frac{1}{2}(3cos^2(\theta) - 1) + A_3 R^3 \frac{1}{2}(5cos^3(\theta) - 3\cos(\theta))$$

$$4kcos^3(\theta) - 3k\cos(\theta) = A_0 - \frac{A_2R^2}{2} + \left[A_1 R - \frac{3}{2}A_3R^3\right]\cos(\theta) + \frac{3A_2R^2}{2}cos^2(\theta) + \frac{5}{2}A_3R^3cos^3(\theta)$$

It is immediately seen that $A_0 = A_2 = 0$ because those terms do not contribute to the value of the potential. Next, compare the terms with similar powers of $\cos(\theta)$ to find the values of the other constants.

$$cos^3(\theta) \text{ terms} \qquad \rightarrow \qquad 4k = \frac{5}{2}A_3R^3$$

$$A_3 = \frac{8k}{5R^3}$$

$\cos(\theta)$ terms,

$$\begin{aligned} -3k &= A_1R - \frac{3}{2}A_3R^3 \\ &= A_1R - \frac{3}{2}\left(\frac{8k}{5R^3}\right)R^3 \\ &= A_1R - \frac{12k}{5} \end{aligned}$$

$$A_1 = \frac{-3k}{5R}$$

$$\begin{aligned} V_{inside}(r,\theta) &= \frac{-3k}{5R}r\cos(\theta) + \frac{8k}{5R^3}\cdot\frac{1}{2}(5cos^3(\theta) - 3\cos(\theta)) \\ &= \frac{-3k}{5R}rP_1(\cos\theta) + \frac{8k}{5R^3}r^3P_3(\cos\theta) \end{aligned}$$

This same method applies to the solution for the potential outside the sphere. Evaluate it at r = R.

$$\begin{aligned} 4kcos^3(\theta) - 3k\cos(\theta) &= \frac{B_0}{R} + \frac{B_1}{R^2}\cos(\theta) + \frac{B_2}{R^3}\left[\frac{1}{2}(3cos^2(\theta) - 1)\right] + \frac{B_3}{R^4}\left[\frac{1}{2}(5cos^3(\theta) - 3\cos(\theta))\right] \\ &= \frac{B_0}{R} - \frac{B_2}{2R^3} + \left[\frac{B_1}{R^2} - \frac{3B_3}{2R^4}\right]\cos(\theta) + \frac{3B_2}{2R^3}cos^2(\theta) + \frac{5B_3}{2R^4}cos^3(\theta) \end{aligned}$$

This shows that B_0 and B_2 must equal zero. The other terms are found from,

$$4k = \frac{5B_3}{2R^4} \qquad \rightarrow \qquad B_3 = \frac{8kR^4}{5}$$

$$\begin{aligned} -3k &= \frac{B_1}{R^2} - \frac{3B_3}{2R^4} \\ &= \frac{B_1}{R^2} - \frac{3}{2R^4}\cdot\frac{8kR^4}{5} \qquad \rightarrow \qquad B_1 = \frac{-3kR^2}{5} \end{aligned}$$

$$
\begin{aligned}
V_{outside}(r,\theta) &= \frac{-3kR^2}{5r^2}\cos(\theta) + \frac{8kR^4}{5r^4}\left[\frac{1}{2}(5cos^3(\theta) - 3\cos(\theta))\right] \\
&= \frac{-3kR^2}{5r^2}P_1(\cos\theta) + \frac{8kR^4}{5r^4}P_3(\cos\theta)
\end{aligned}
$$

Find the surface charge density, $\sigma_0(\theta)$

This can always be found as the discontiniuty in the electric field at the surface (Equation 3.82 in Girffiths), but in this particular problem it is more easily solved with,

$$
\frac{\sigma_0(\theta)}{\epsilon_0} = \sum_{l=0}^{\infty}(2l+1)A_l R^{l+1} P_l(\cos\theta) \qquad \text{Eq. 3.83}
$$

¿From our previous work in this problem we already know that the term on the right hand side contains only the l = 1 and l = 3 terms. Combining this with the values of A_1 and A_3 we have found (we can leave the Legendre polynomials alone, there is no reason to write them out),

$$
\begin{aligned}
\frac{\sigma_0(\theta)}{\epsilon_0} &= (3A_1P_1(\cos\theta)) + (7A_3R^2P_3(\cos\theta)) \\
&= 3\left(\frac{-3k}{5R}\right)P_1(\cos\theta) + 7R^2\left(\frac{8k}{5R^3}\right)P_3(\cos\theta) \\
&= \frac{-9k}{5R}P_1(\cos\theta) + \frac{56k}{5R}P_3(\cos\theta)
\end{aligned}
$$

$$
\sigma_0(\theta) = \epsilon_0\left[\frac{-9k}{5R}P_1(\cos\theta) + \frac{56k}{5R}P_3(\cos\theta)\right]
$$

[5] Problem 3.20 from Griffiths

Find the potential outside a metal sphere of total charge Q placed in a uniform electric field, $\vec{E}_0$. The radius of the sphere is R. Clearly explain where you are setting the reference (i.e. zero) of your potential.

As the argument from our Discussion Section on February 9 went, we may treat this problem as a point charge of value Q centered at the origin in addition to a neutral sphere placed in the uniform background field. To recap, when we have a neutral sphere the background field will polarize it (i.e. some positive charge accumulates at one end of the sphere and an equal amount of negative charge accumulates at the other end). Since this is a conducting sphere we know that it is an equipotential and that there is no electric field inside it. This means that we can now add a charge Q to the sphere and it will disperse itself evenly across the surface, just as if there were no background field or polarization.

Since we are only looking for the potential outside the sphere we know that the contribution to the potential due to this uniform charge distribution is simply that due to a point charge located at the origin,

$$V_Q(r,\theta) = \frac{Q}{4\pi\epsilon_0 r}$$

We are left to find the contributions to the potential due to a neutral sphere in a background electric field. An illustration of this situation is given as Figure 3.24 in Griffiths. The first item in solving this problem is to determine where the reference point should be set. There is a uniform background electric field that stretches to infinity so we can't set the reference at $r = \infty$. Think of the three terms we expect to find in the final solution for the potential.

(1) That due to the equivalent point charge, Q, at the origin.
(2) That due to the uniform background field.
(3) That due to the surface charge distribution of the neutral sphere after it is placed in the uniform background field.

The reference point should be set according to a convention that is allowed by all of the contributions above. (1) tells us that the reference should occur at infinity. (2) tells us we can set any plane perpendicular to the direction of $\vec{E}$ as our reference (see Griffiths' Example 9). This means that (3) is the contribution that is limiting our reference choice. Since there are charges on two ends of the sphere it is logical (though by no means obvious) to choose a slice of the sphere separating the resulting halves as the reference. Take the plane between the positive and negative charges and call the potential there to be zero. If you go toward the positive charges you see a greater potential and if you approach the negative charges you see a negative potential.

Deciding on a reference that satisfies all of the requirements set above leaves us with only one choice. The potential is set to be zero at $r = \infty$ in the xy-plane. The potential is non-zero in the xy-plane just outside the sphere due to the net charge, Q, but as you stay in this plane and move ever farther from the sphere your potential goes to zero.

The contributions to the total potential that come from the neutral sphere in a uniform background electric field are found in Example 3.9,

$$V_{other}(r,\theta) = -E_0 r\cos(\theta) + \frac{E_0 R^3}{r^2}\cos(\theta) \qquad \text{Eq. 3.76}$$

The final solution for the potential in this situation is,

$$\boxed{V(r,\theta) = -E_0 r\cos(\theta) + \frac{E_0 R^3}{r^2}\cos(\theta) + \frac{Q}{4\pi\epsilon_0 r}}$$